Last updated: Yeh-Liang Hsu (2010-10-29).

Note: This is the course material for "ME550 Geometric modeling and computer graphics," Yuan Ze University. Part of this material is adapted from *CAD/CAM Theory and Practice*, by Ibrahim Zeid, McGraw-Hill, 1991. This material is be used strictly for teaching and learning of this course.

Analytic surfaces

1. Surface models

Shape design and representation of complex objects such as car, ship and airplane bodies as well as castings cannot be achieved utilizing wireframe modeling. In such cases, surface modeling must be utilized to describe objects precisely and accurately. Due to the richness in information of surface models, their use in engineering and design environments can be extended beyond just geometric design and representation. They are usually used in various applications such as calculating mass properties, checking for interference between mating parts, generating cross-sectioned views, generating finite element meshes, and generating NC tool paths for continuous path machining.

A surface model of an object is a more complete and less ambiguous representation that its wireframe model. Surface models take the modeling of an object one step beyond wireframe models by providing information on surfaces connecting the object edges. Visualization of a surface is aided by the addition of artificial fairing lines (called mesh), which criss-cross the surface and so break it up into a network of interconnected patches. Surface models provide hidden line and surface algorithms to add realism to the displayed geometry. Shading algorithms are only available for surface and solid models.

There are two types of surfaces. Analytic surfaces are based on wireframe entities, and include the plane surface, ruled surface, surface of revolution, and tabulated cylinder. Synthetic surfaces are formed from a given set of data points or curves and include the bicubic, Bezier, B-spline, and Coons patches.

Plane surface

This is the simplest surface. It requires three non-coincident points to define an infinite plane. The plane surface can be used to generate cross-sectional views by intersecting a surface model with it, generate cross sections for mass property calculations, or other similar applications where a plane is needed. Figure 1 shows a plane surface.

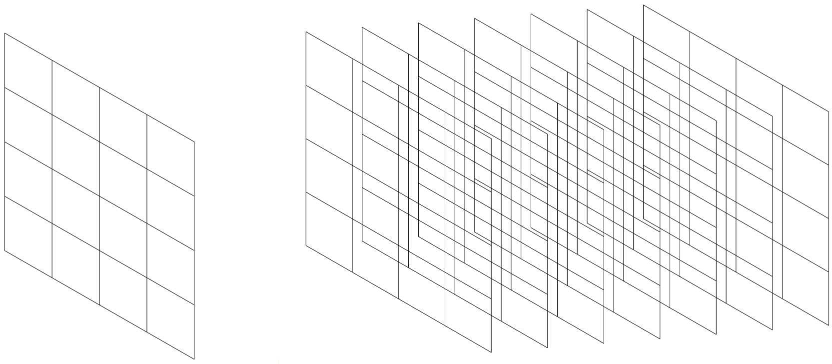


Figure 1. Plane surface

Ruled (lofted) surface

This is a linear surface. As shown in Figure 2, a ruled surface interpolates linearly between two boundary curves that define the surface (rails). Rails can be any wireframe entity.

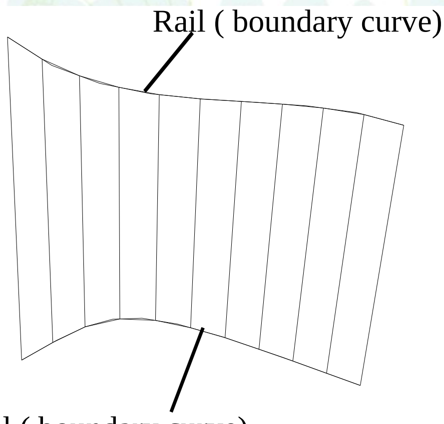


Figure 2. Ruled surface

Surface of revolution

This is an axisymmetric surface that can model axisymmetric objects. It is generated by rotating a planar wireframe entity in space about the axis of symmetry a certain angle (Figure 3).

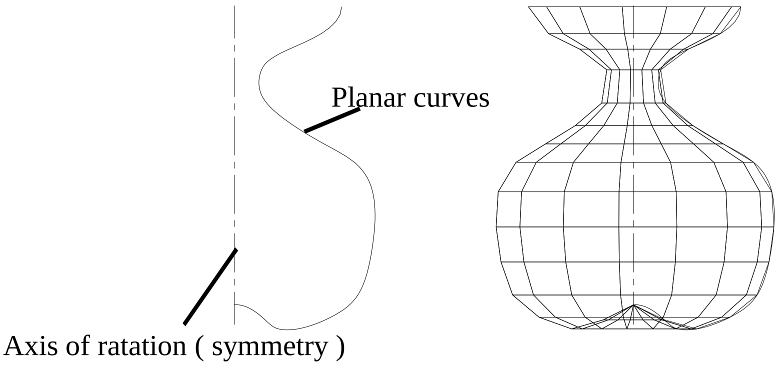


Figure 3. Surface of revolution

Tabulated cylinder

This is a surface generated by translating a planar curve a certain distance along a specified direction (axis of the cylinder) as shown in Figure 4.

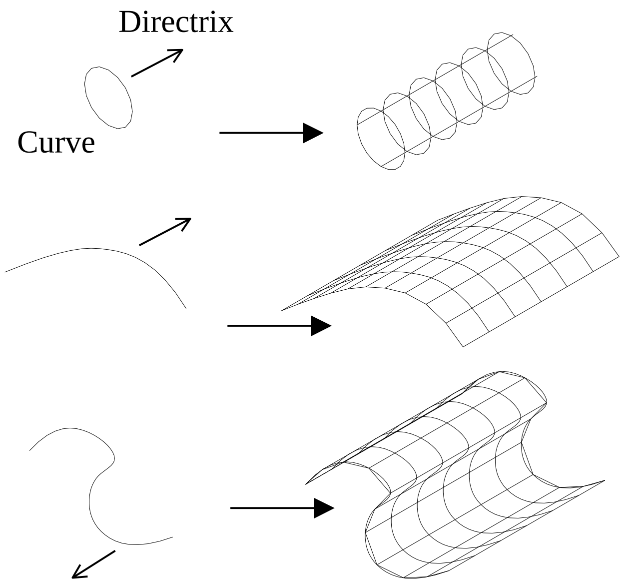


Figure 4. Tabulated cylinder

◇Assignment 1

Construct the analytical surfaces discussed above using your CAD software, if applicable. Describe the procedure for constructing these surfaces. ◇

Bezier surface

This is a surface that approximates given input data (Figure 5). It is different from the previous surfaces in that it is a synthetic surface, it does not pass through all given data points.

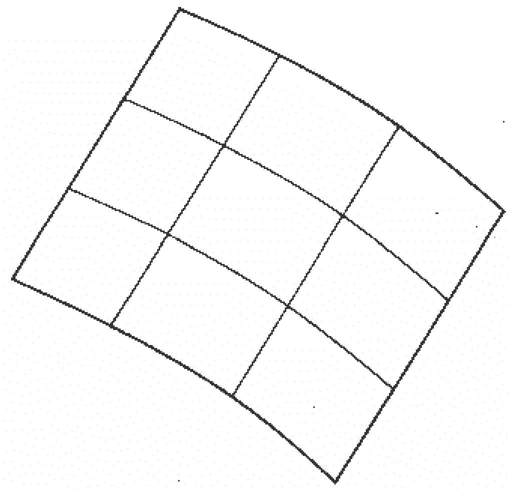
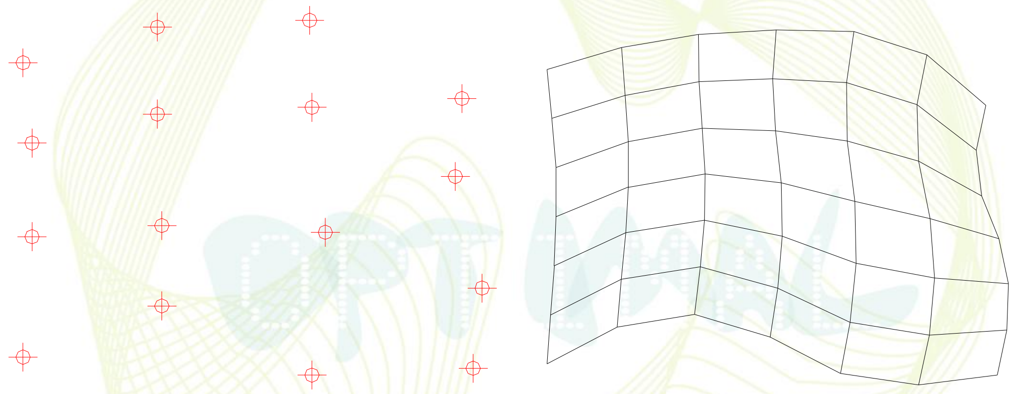


Figure 5. Bezier surface

B-spline surface

This is a surface that can approximate or interpolate given input data (Figure 6). It is a synthetic surface. It is a general surface like the Bezier surface but with the advantage of permitting local control of the surface.

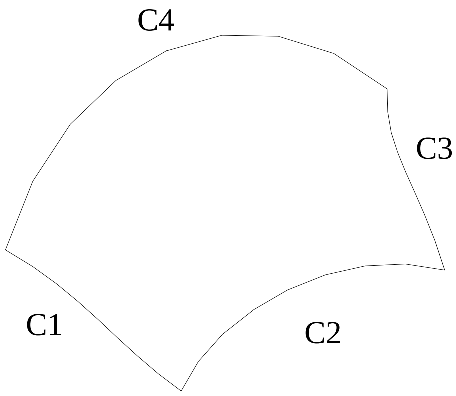


(a) Data points (b) B-spline

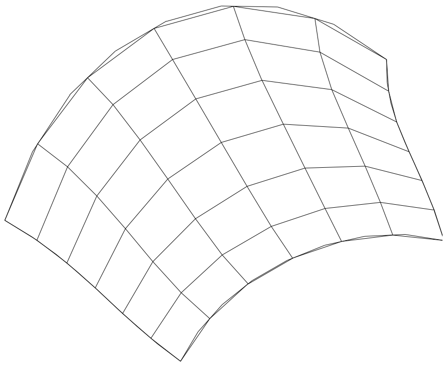
Figure 6. B-spline surface.

Coons patch

The above surfaces are used with either open boundaries or given data points. The Coons patch is used to create a surface using curves that form closed boundaries (Figure 7).



(a) Closed boundary



(b) Coons patch

Figure 7. Coons patch

◇Assignment 2

Construct the synthetic surfaces discussed above using your CAD software, if applicable. Describe the procedure for constructing these surfaces. ◇

Fillet surface

This is a B-spline surface that blends two surfaces together (Figure 8).

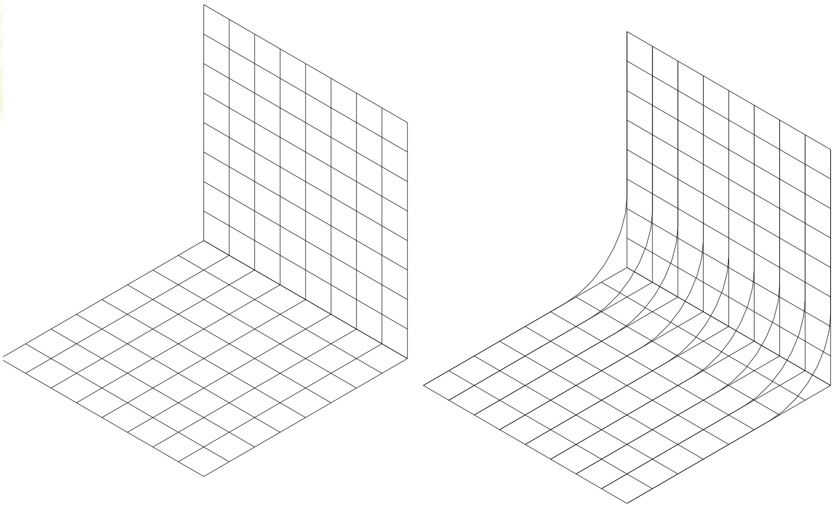


Figure 8. Fillet surface.

Offset surface

Existing surfaces can be offset to create new ones identical in shape but may have different dimensions. For example, as shown in Figure 9, to create a hollow cylinder, the outer or inner cylinder can be created using a cylinder command and the other one can be created by an offset command.

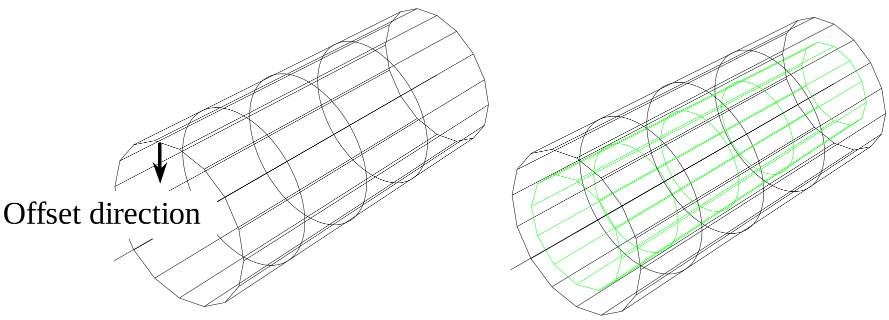


Figure 9. Offset surface

◇Assignment 3

Construct the fillet and offset surfaces discussed above using your CAD software, if applicable. Describe the procedure for constructing these surfaces. Are there other special surfaces supported by your CAD software? Describe these special surfaces. ◇

2. Parametric representations of analytic surfaces

Surface equations, like curve equations, can be classified into parametric representations and nonparametric representations. Consider a sphere of radius R centered at the origin of the reference coordinate system. The sphere can be expressed as

$$x^2 + y^2 + z^2 - R^2 = 0 \quad (1)$$

The parametric representation of the sphere will be

$$\mathbf{P}(u,v) = [x \quad y \quad z]^{\mathrm{T}} = [R\cos u \cos v \quad R\sin u \cos v \quad R\sin v]^{\mathrm{T}},$$

$$0 \le u \le 2\pi, 0 \le v \le 2\pi \quad (2)$$

In general, two parameters are required to represent a surface,

$$\mathbf{P}(u,v)=[x \quad y \quad z]^{\mathrm{T}}=[x(u,v) \quad y(u,v) \quad z(u,v)]^{\mathrm{T}},$$

$$u_{\min}\le u\le u_{\max}, v_{\min}\le v\le v_{\max} \tag{3}$$

This parametric equation uniquely maps the parametric space (E^2 in *u* and *v* values) to the Cartesian space (E^3 in *x*, *y*, and *z*). To generate curves on a surface patch, one can fix the value of one of the parametric variables, say *u*, to obtain a curve in terms of the other variable, *v*. The tangent vector at any point **P**(*u*, *v*) on the surface is obtained by holding one parameter constant and differentiating with respect to the other. Therefore, there are two tangent vectors.

$$\begin{bmatrix}\mathbf{P}_u\\ \mathbf{P}_v\end{bmatrix}=\begin{bmatrix}\frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u}\\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v}\end{bmatrix} \tag{4}$$

Parametric representations of analytic surfaces of analytic surface are discussed as follows:

Plane surface

A plane defined by three points $\mathbf{P}_0$, $\mathbf{P}_1$, and $\mathbf{P}_2$ can be expressed as

$$\mathbf{P}(u,v)=\mathbf{P}_0+u(\mathbf{P}_1-\mathbf{P}_0)+v(\mathbf{P}_2-\mathbf{P}_0), \quad 0\le u\le 1, 0\le v\le 1 \tag{5}$$

Ruled surface

A ruled surface is generated by joining corresponding points on two space curves (rails) **G**(*u*) and **Q**(*u*) by straight lines (also called rulings or generators), every developable surface is a ruled surface.

$$\mathbf{P}(u_i,v)=\mathbf{G}_i+v(\mathbf{Q}_i-\mathbf{G}_i) \tag{6}$$

$$\mathbf{P}(u,v)=\mathbf{G}(u)+v[\mathbf{Q}(u)-\mathbf{G}(u)]=(1-v)\mathbf{G}(u)+v\mathbf{Q}(u), \quad 0\le u\le 1, 0\le v\le 1 \tag{7}$$

A ruled surface can only allow curvature in the *u* direction of the surface provided that the rails have curvatures. The surface curvature in the *v* direction (along the rulings) is zero and thus a ruled surface cannot be used to model surface patches that have curvatures in two directions.

◇Assignment 4

Assume the coordinates of control points of two cubic Bezier curves **G**(u) and **Q**(u). Write a Matlab program to construct a ruled surface using Equation (7). Display the surface by drawing meshes for v=0, v=1, u=0, u=0.2, u=0.4, u=0.6, u=0.8, u=1. Show your Matlab program too. ◇

◇Assignment 5

For the surface you generated in Assignment 4, calculate the tangent vectors at the point u=0.5, v=0.5. ◇

Surface of revolution

The rotation of a planar curve an angle v about an axis of rotation creates a circle (if v=360) for each point on the curve whose center lies on the axis of rotation and whose radius $r_z(u)$ is variable, as shown in Figure 11. The planar curve and the circles are called the profile and parallels respectively while the various positions of the profile around the axis are called meridians.

$$\mathbf{P}(u,v) = r_z(u)\cos v\hat{\mathbf{n}}_1 + r_z(u)\sin v\hat{\mathbf{n}}_2 + r_L(u)\hat{\mathbf{n}}_3, \quad 0 \le u \le 1, 0 \le v \le 2\pi \qquad (8)$$

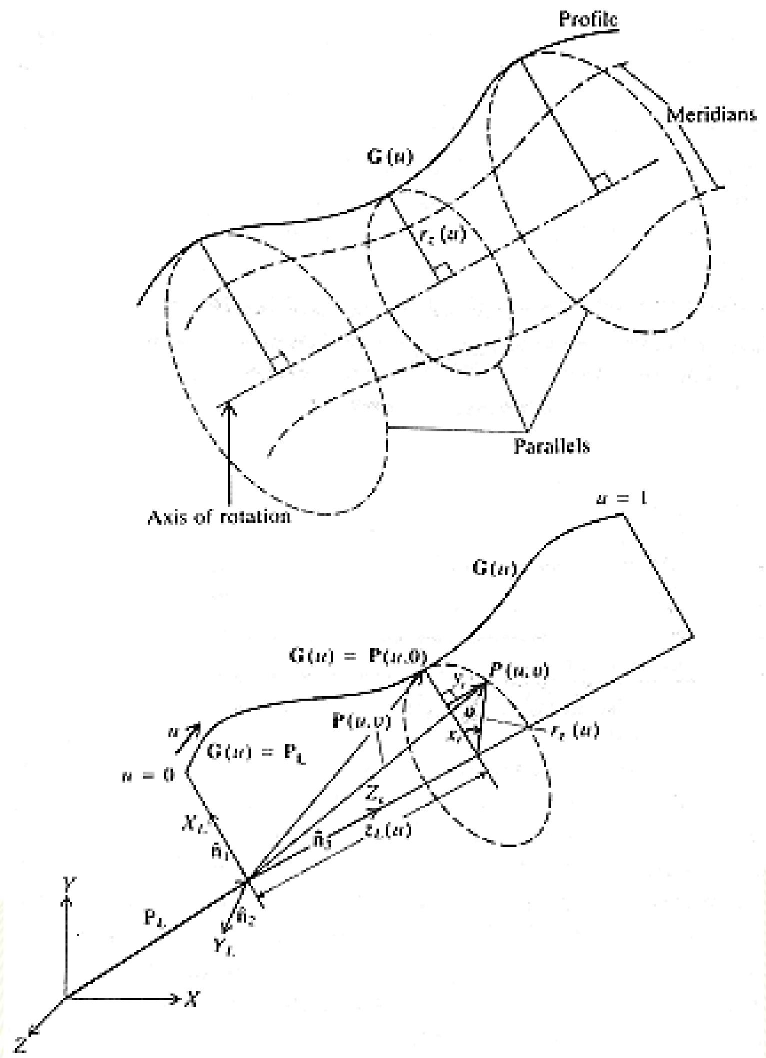


Figure 11. Parametric representation of a surface of revolution.

◇Assignment 6

Assume the coordinates of the control points and create a cubic Bezier curve $r_z(u)$. Let $\hat{\mathbf{n}}_1 = \hat{\mathbf{x}}$, $\hat{\mathbf{n}}_2 = \hat{\mathbf{y}}$, and $\hat{\mathbf{n}}_3 = \hat{\mathbf{z}}$, rewrite Equation (8). What is $r_L(u)$ in this case? Write a Matlab program to plot this surface. Use proper mesh and view to display the surface. Show your Matlab program too. ◇

Tabulated Cylinder

A tabulated cylinder has been defined as a surface that results from translating a space planar curve along a given direction.

$$\mathbf{P}(u,v) = \mathbf{G}(u) + v\hat{\mathbf{n}}_v, \quad 0 \le u \le u_{\max}, 0 \le v \le v_{\max} \tag{9}$$