A fuzzy optimization algorithm for the blow moulding process

This paper presents recent results of an on-going NRC-NSC joint research project on the development of a multidisciplinary design optimization (MDO) methodology for blow moulded automotive parts. In particular, this paper demonstrates a fuzzy optimization algorithm for determining the optimal gap openings and die geometry in the blow moulding process. Traditional numerical optimization algorithms treat the optimization problem as pure mathematical problems. Engineering knowledge about the problem is not utilized in the optimization process. The idea of the fuzzy optimization algorithm is that, instead of using purely numerical information to obtain the new design point in the next iteration, engineering knowledge and human supervision process can be modelled in the optimization algorithm using fuzzy rules. It is shown that how a single fuzzy optimization engine can be used in various types of optimizations.

Key Words: blow moulding, process optimization, fuzzy logic

1. Introduction

Blow moulding is the forming of a hollow part by “blowing” a mould-cavity-shaped parison that is made by thermoplastic molten tube. Blow moulding is the most popular and efficient process for manufacturing commodity hollow plastic parts such as bottles, containers, toys, etc. More recently, this forming process has been applied to the manufacture of complex automotive parts such as fuel tanks, seat backs, air ducts, windshield washer and cooling reservoirs.
The blow moulding process consists of three phases: parison extrusion, part inflation and part solidification. The extrusion phase involves the extrusion of a polymer melt through an annular die to form a hollow cylindrical parison with a non-uniform material distribution and consequently non-uniform parison thickness along its length. Once the parison is extruded to the desired length, it is inflated to take the shape of an enclosing mould. The part then solidifies as a consequence of the heat transfer to the cooling mould. The parison thickness distribution is modified significantly by the inflation and the solidification stages to yield the final part thickness distribution.

Blow moulded parts often require a strict control of the thickness distribution in order to achieve the required mechanical performance and final weight. Manipulation of the die gap programming points can lead to an optimal part thickness distribution. Figure 1 shows the forming of an axis symmetric bottle. As illustrated in Figure 1(a), the die gap can be adjusted as a function of time in order to obtain the desired thickness profile along the extruded parison, which determines the thickness of the blown hollow part. For example, in order to obtain uniform thickness distribution of the hollow part, the thickness of programmed parison must be non-uniform. As shown in Figure 1(b), the parison thickness for the greatest expansion area must be thicker than those of the other areas.

BlowSim is a finite element software package designed to simulate the extrusion blow moulding, injection stretch blow moulding, and thermoforming processes. It is developed by the Industrial Material Institute (IMI) of National Research Council (NRC), Canada. The blow moulding process simulation consists of the modelling of the successive process stages in
order to predict the final part quality as a function of the operating conditions, the mould geometry and the material properties. BlowSim can be used to model the process phases: parison formation, clamping and inflation, part cooling and shrinkage, and part mechanical performance. The process modelling is based on a large displacement finite element formulation [Laroche et al., 1999]. The parison deformation is modelled using a multi-layer membrane element type and a non-isothermal visco-elastic material model. The mechanical performance is modelled with the predicted thickness distribution, and the appropriate applied load. The simulation results of BlowSim have been validated with many industrial cases and show good agreement.

In many industrial applications, combining simulation tools with optimization methodologies allows the designers to treat complex design criteria via simulation to pursue the maximum part quality and minimum manufacturing costs. In the blow moulding process, it is desirable to manipulate the die gap programming to obtain a final part of constant thickness or a predefined thickness profile. It is, therefore, an optimization problem on how to control the gap openings to minimize the deviation of the thickness of the final part from the target thickness. Given a set of gap openings, we are able to extract the thickness of all nodes from the simulation results by BlowSim, and apply them into the following equation to get the objective function value:

\[
\text{min. } f = \left( \frac{\sum_{i=1}^{n} (y_i - Y_i)^2}{n-1} \right)^{0.5}
\]  

(1)

where \(y_i\) is the thickness at the \(i\)-th node in the simulation model, \(Y_i\) is the corresponding target thickness, and \(n\) is the total number of nodes. The gap-openings at discrete time points are the design variables. Obviously, \(y_i\) is a function of the gap openings.

Gradient type numerical optimization algorithms provide a numerical tool to solve for the optimal gap openings. On the other hand, manufacturing engineers usually adjust the gap openings empirically: reduce the gap opening if the corresponding portion of final part is too thick, and vice versa.

When solving an engineering optimization problem using numerical optimization algorithms, we basically view the problem as a pure mathematical optimization model. Design modifications in the optimization process rely on numerical information rather than engineering heuristics, experiences, and knowledge. This paper develops a “fuzzy optimization algorithm” for engineering optimization problems, which enables the use of engineering heuristics to generate the new design point of the next iteration. The structure of an optimization algorithm is still maintained to guide the engineering decision process and to
ensure an optimal solution can be obtained. Currently this fuzzy optimization algorithm is developed specifically for engineering optimization problems whose objective functions are in the form of Equation (1).

This paper first explains the concept of fuzzy optimization algorithms. The blow moulding process optimization results are presented to demonstrate that this approach can be general to various optimization cases in different application domains.

2. The concept of “fuzzy optimization algorithms”

As shown in Figure 2, the optimization process can be viewed as a closed-loop control system. The optimization model in an optimization process is analogous to the plant in a control system; an optimization algorithm is analogous to the controllers. Initial parameters are input to the optimization algorithm, which in turn generates a trial design point according to its search rules. The optimization model is then evaluated at this trial design point, and the information such as objective and constraint function values and sensitivity is fed back. The optimization algorithm is triggered again to generate the next design point, using the information from previous iterations. Finally, a control system attempt to achieve a stable, predefined output. The optimization process pursues a converging objective function value.

**Figure 2. General block diagram of a design optimization process**

Traditional numerical optimization algorithms are analogous to direct digital controllers. The algorithms are usually “crisply” designed for well defined mathematical models. Their numerical rules for generating the next design point are exact and definite, and they can usually be proved to have nice converging behavior when applying to well defined mathematical models. However, in engineering optimization problems, we seldom have well defined mathematical models. The functions in an engineering optimization problem often do not have exact algebraic forms in terms of the design variables, and they can only be evaluated through experiments or computer simulations, which are expensive and imprecise in nature. Sensitivity required in most numerical optimization algorithms is often obtained from finite
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difference methods. Very often the cost of the number of function evaluations required to meet the “crisp” definition of the numerical algorithms is simply too high to be affordable.

When we apply numerical optimization algorithms on an engineering problem, we treat the engineering problem as a pure mathematical problem. Engineering heuristics are totally ignored. This motivates the idea that, in addition to crisp numerical rules, the human supervision process should also be modelled in an optimization algorithm using fuzzy rules. As suggested in Figure 2, the “controllers” in the optimization process may as well be fuzzy controllers!

A fuzzy system is characterized by a collection of linguistic statements based on expert knowledge. The linguistic statements are usually in the form of IF-THEN rules. As shown in Figure 2, if the relations between the system process input $x^q$ (gap openings) and system process output $y^q$ (thickness in the blow moulding example) and $\Delta y^q$ are known empirically (reduce the gap opening will reduce the thickness of the corresponding portion of the final part, and vice versa), the fuzzy logic optimization engine will generate the system process input change rate $\Delta x^q$ according to a set of domain parameters given by the users. The step size $\alpha^q$ in $\Delta x^q$ is set to be 1 initially, but is also controlled by the same fuzzy optimization engine. The system process input is then updated ($x^{q+1} = x^q + \Delta x^q$) and the new system process outputs are fed back to compare with set point $Y$. This iterative process continues until the predefined convergence criterion is met.

Hsu et al [1995] proposed this concept of a fuzzy optimization algorithm, and Mulkay and Rao [1998] also presented the same idea. In both work, fuzzy heuristics are used to control the parameters of the optimization algorithm to improve its performance. The following sections demonstrate how engineering heuristics are modelled into the fuzzy optimization algorithm for the optimization of the blow moulding process.

3. The blow moulding examples for constant thickness

3.1 The bottle example

The bottle example in Figure 1 is first used to illustrate the fuzzy optimization process. In this example, we hope to manipulate the gap openings at 7 control points to obtain a constant thickness part at 2mm. Therefore, in the objective function Equation (1), $Y_i = 2$, and $n = 7$. As discuss earlier, manufacturing engineers usually adjust the gap openings empirically: reduce the gap opening if the corresponding portion of final part is too thick, and vice versa. This engineering heuristic indicates that the thickness of a certain node ($y_i$) is a monotonic
increasing function with respect to the corresponding gap opening \((x_j)\), and can be expressed by 5 fuzzy rules:

1. IF \(y_i\) is PB THEN \(\Delta x_i\) is NB;
2. IF \(y_i\) is PS THEN \(\Delta x_i\) is NS;
3. IF \(y_i\) is ZE THEN \(\Delta x_i\) is ZE;
4. IF \(y_i\) is NS THEN \(\Delta x_i\) is PS;
5. IF \(y_i\) is NB THEN \(\Delta x_i\) is PB.

The quantization table (Table 1) gives quantitative definitions for PB (positive big), PS (positive small), ZE (zero), NB (negative small) and NB (negative big). There are 5 “domain parameters” in Table 1 to be decided by the user according to the application problem. From BlowSim simulation, when the gap openings at the 7 control points are all set to be 75%, the maximum thickness of the part is 6.05mm, and the minimum thickness of the part is 1.36mm. Therefore, the definitions of the 5 domain parameters and their values for the bottle example are

\(Y_i\): Target value of system process output (target thickness, 2mm);

\(y_{i,\text{max}}\): Maximum value of system process output \(y_i\) (maximum thickness in the initial design, 6mm);

\(y_{i,\text{min}}\): Minimum value of system process output \(y_i\) (minimum thickness in the initial design, 1mm);

\(x_{i,\text{max}}\): Maximum value of system process input \(x_i\) (maximum allowable gap opening, 95%);

\(x_{i,\text{min}}\): Minimum value of system process input \(x_i\) (minimum allowable gap opening, 5%).
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Table 1. The quantization table

<table>
<thead>
<tr>
<th>Boundaries of fuzzy input, $y_i$</th>
<th>Boundaries of fuzzy output, $\Delta x_i$</th>
<th>Quantized Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_i + (Y_{i, \text{max}} - Y_i)/Q_{is}$</td>
<td>$x_{i, \text{max}} - x_i$</td>
<td>2</td>
</tr>
<tr>
<td>$Y_i - (Y_{i, \text{min}} - Y_i)/Q_{os}$</td>
<td>$(x_{i, \text{min}} - x_i)/Q_{os}$</td>
<td>1</td>
</tr>
<tr>
<td>$Y_i$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Y_i - (Y_{i, \text{min}} - Y_i)/Q_{os}$</td>
<td>$(x_{i, \text{min}} - x_i)/Q_{os}$</td>
<td>1</td>
</tr>
<tr>
<td>$Y_i - (Y_{i, \text{min}} - Y_i)/Q_{os}$</td>
<td>$x_{i, \text{min}} - x_i$</td>
<td>2</td>
</tr>
</tbody>
</table>

In this research, the process of assigning the values of the 5 domain parameters to the current application problem is called “domain parameters mapping.” There are two process parameters $Q_{is}$ and $Q_{os}$ in Table 1, which define the linearity of quantized level with respect to the fuzzy input and fuzzy output. This relation is linear when $Q_{is}=2$, and both $Q_{is}$ and $Q_{os}$ are set to be 2 through out this paper.

In the blow moulding process simulation, BlowSim provides the “average weighted thickness” of all nodes affected by the gap opening of a certain control points. Given a set of gap openings, we are able to extract the average weighted thickness of all control points from the simulation results by BlowSim. The fuzzy optimization engine will then generate a set of change in gap openings $\Delta x_i$ for the next iteration according to the current average weighted thickness and the domain parameters defined by the user.

Referring to Figure 2, the step size $\alpha$ is set to be 1 initially, but is also controlled by the same fuzzy optimization engine. In the optimization iterations, we expect a converging behavior in the objective function. However, if the step size $\alpha$ is too big, the objective function value might “overshoots.” On the other hand, if the step size $\alpha$ is too small, the convergence will be slow. Ideally, step size $\alpha$ should be adjusted dynamically through out the iteration process, and the heuristic rule for adjusting $\alpha$ is simply, reduce $\alpha$ if the change in objective function value is big, and vice versa. Obviously this can also be expressed by the same 5 fuzzy rules previously discussed, only now fuzzy input $y_i = (f_k - f_{k-1})/f_{k-1}$, where $f_k$ is the objective function at $k$-th iteration, and fuzzy output $\Delta x_i$ becomes change in step size $\Delta r$. The step size for the $(k+1)$-th iteration is, $\alpha_{k+1} = (1 + \Delta r) \cdot \alpha_k$. Same domain parameter mapping is also required here: $Y_i=0$ (we expect no change in objective function value when converging), $y_{i, \text{max}}=10\%$, $y_{i, \text{min}}=-10\%$, $x_{i, \text{max}}=1.0$, and $x_{i, \text{min}}=0.5$ (step length in the next iteration will be $0.5\sim1.0$ times of that of the previous iteration). These parameters are used for step size control through out this paper.
Finally, Figure 4(a) shows the iteration history of the bottle example, and Figure 4(b) compares the gap openings of the initial and final design. Figure 4(c) compares the average weighted thickness of the initial and final design on the 7 control points, and Figure 4(d) compares the thickness distribution of the initial and final parts.

In this example, the optimization process terminates after 18 iterations, when the change in objective function value is less than 0.1%. Only 18 BlowSim simulations are needed, and sensitivity calculation is not required. Ideally the objective function should converge to zero if a constant thickness part is obtained. However, as shown in Figure 4(c), the thickness of the top and bottom portions of the bottle are higher than the target value. As shown in Figure 4(b), the thickness at these two portions cannot be further reduced because the corresponding gap openings are already close to the lower bound 5%. Figure 5 shows the optimization result using 31 control points. Increasing the resolution of the control points further reduces the objective function value.
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Figure 4. Optimization results of the bottle example with 7 control points
3.2 Fluid reservoir and gas tank example

The fuzzy optimization algorithm is then used to for process optimization of two complex automotive parts, the fluid reservoir and the gas tank. In the fluid reservoir example, the target thickness is 5mm. In the initial design with 23 control points, the maximum thickness of the part is 6.7009mm, and the minimum thickness of the part is 2.3885mm. With these information, the values of the domain parameters in this case are assigned as follow: \( Y_i = 5 \text{mm}, y_{i,\text{max}} = 7 \text{mm}, y_{i,\text{min}} = 2 \text{mm}, x_{i,\text{max}} = 95\%, \) and \( x_{i,\text{min}} = 5\% \).

The optimization process terminates after 15 iterations. Figure 6 shows the optimization results. In Figure 6(c), the weighted average thickness of all control points are close to 5mm, but the objective function value is still higher than 0. This is because the fluid reservoir is not symmetric. It is not possible to obtain a constant thickness part using a circular die. Die geometry optimization is needed here, and will be discussed in the later sections.

In the gas tank example, the target thickness is also 5mm, and 20 control points are used. With the information from the simulation of the initial design, the values of the domain parameters in this case are assigned as follow: \( Y_i = 5 \text{mm}, y_{i,\text{max}} = 13 \text{mm}, y_{i,\text{min}} = 4 \text{mm}, x_{i,\text{max}} = 95\%, \) and \( x_{i,\text{min}} = 5\% \). The optimization process terminates after 27 iterations, though a better objective function value has been obtained in the 3\textsuperscript{rd} iteration. Figure 7 shows the optimization results. Again, in Figure 7(c), the weighted average thickness of all control points are close to 5mm, but the objective function value is still high. And when the average weighted thickness approaches 5mm the thickness of all nodes do not necessarily approach 5mm. This is also
because the gas tank is not symmetric. Figure 7(b) shows that the gap openings of the first 8 nodes have reached the lower bound, which also prevents the objective function value to go down further.

Figure 6. Optimization results of the fluid reservoir example
4. Performance optimization

Another major requirement of blow moulded parts is its mechanical performance. A common practice for achieving this goal is to minimize the part weight subject to mechanical performance constraints such as maximum stress and part deflection under top load, internal pressure, etc. Figure 8(a) shows the bottle example again. Two loading conditions are considered separately, top load and internal pressure. The maximum stress (from finite element software) under these two loads $\text{Max. } (\sigma_{\text{topload}}, \sigma_{\text{pressure}})$ shall not exceed a predefined value. In order to minimize the part weight, it is desirable to find an optimal thickness profile that is fully stressed under these two loads, that is, $\text{Max. } (\sigma_{\text{topload}}, \sigma_{\text{pressure}})$ equals the maximum allowable stress at all nodes. This is again an optimization problem whose objective function
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is in the form of Equation (1). The engineering heuristic is very similar too: increase the thickness if the stress is too high and vice versa. The same fuzzy optimization algorithm can be used to find the optimal thickness profile, which is also described by control points. Now fuzzy input $y_i$ is the Max. ($\sigma_{\text{topload}}$, $\sigma_{\text{pressure}}$) on the $i$-th control point, and fuzzy output $\Delta x_i$ becomes change in thickness on the $i$-th control point. The initial design in this case is 2mm constant thickness. The values of the domain parameters in this case are assigned as follow: $Y_i=16.5\text{MPa}$ (target stress, yield stress/2); $y_{i,\text{max}}=33\text{MPa}$ (yield stress); $y_{i,\text{min}}=10\text{MPa}$ (low stress in the initial design), $x_{i,\text{max}}=4\text{mm}$ (thickness upper bound), and $x_{i,\text{min}}=1\text{mm}$ (thickness lower bound). The optimization process terminates after 8 iterations (Figure 8(b)). When we superimposes the stress distributions of two loads for the final design in Figure 8(c), we can see that constant stress is achieved in almost all control points. Figure 8(d) shows the final thickness profile.
This optimal thickness profile can be used as target thickness to obtain the gap openings that achieve this thickness profile. As shown in, Figure 9(a) shows the resulting thickness profile obtained using 7 control points, which do not agree very well with the optimal thickness profile. Figure 9(b) shows the stress distribution of the final design. An alternative is to directly use gap openings at the control points as design variables, that is, to find the optimal gap openings that achieve the constant stress boundary. The engineering heuristic is, reduce the gap opening if the stress is low, and vice versa. The values of the domain parameters in this approach are: $Y_i=16.5\text{MPa}$; $y_{i,\max}=33\text{MPa}$; $y_{i,\min}=10\text{MPa}$, $x_{i,\max}=95\%$, and $x_{i,\min}=5\%$. Figure 10 shows the results of this approach.

At this point, both results are not good. The resolution using 7 control points is not enough to represent a complex shape in Figure 8(d) seems to be the major problem. This example will be redone with more control points.
5. Die geometry optimization

As discussed earlier, in some cases, especially for unsymmetrical parts, die geometry has to be manipulated in order to obtain desired part thickness. The geometry of the die in the closed and open positions is defined by the minimum and maximum die gap at a number of "die points" in different angular positions. The minimum and maximum die gap at each die point (and of course gap openings) can be manipulated to obtain a constant thickness part.
BlowSim also provides the average weighted die point thickness (same philosophy as average weighted thickness previously used for gap openings).

The first trial for die geometry optimization will be to manipulate only the maximum die gap, while keeping the gap openings at the "optimal levels" obtained previously. The domain parameters mappings for the gas tank example in this case are: $Y_i = 5\text{mm}$, $y_{i,\text{max}}=50\text{mm}$, $y_{i,\text{min}}=[2.8 \ 8.4 \ 11.2 \ 9.8 \ 2.8 \ 5.6 \ 11.2 \ 7.0 \ 2.8]\text{mm}$, $x_{i,\text{max}}=10\text{mm}$, and $x_{i,\text{min}}=0.5\text{mm}$. Figure 11(a) shows the iteration history of the bottle example, and Figure 11(b) compares the maximum die gaps of the initial and final design. Figure 11(c) compares the average weighted die point thickness of the initial and final design on the 9 die points, and Figure 11(d) compares the thickness distribution of the initial and final parts. The results are not satisfactory at this point. Figure 11(a) does not show any sign of convergence, though Figure 11(a) shows the average weighted thickness is closer to target 5mm than the initial design.
6. Author Biography

Dr. Yeh-Liang Hsu received his PhD from Stanford University in 1992. He is professor and chairman of ME Department, Yuan Ze University, Taiwan. He specializes in design optimization.

Tzu-Chi Liu is a 2nd year PhD student.

7. References

